Autoseg 2013
Opening remarks

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Dice's coefficient

- Invented in 1940's for botany applications

\[ Dice(A, B) = \frac{2 |A \cap B|}{|A| + |B|} \]
Dice's coefficient

- Invented in 1940's for botany applications

\[
\text{Dice}(A, B) = \frac{2|A \cap B|}{|A| + |B|}
\]
Dice's coefficient

• What is the Dice coefficient when matching a parotid gland with a sphere?
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Dice's coefficient

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Dice's coefficient

- What is the Dice coefficient when matching a parotid gland with a sphere?

Dice coefficient = 0.74
WARNING

Professional Driver. Closed course.

Photo credit: http://nytimes.com
Average Dice

- Average the Dice over all structures of interest

\[
\sum_{S \in \text{structures}} \text{Dice}(S, \hat{S})
\]
Average Dice

![Average Dice](chart.png)

- **Y-axis**: Left Parotid Dice
- **X-axis**: Brainstem Dice

The scatter plot shows the distribution of Left Parotid Dice against Brainstem Dice.
Hausdorff distance

\[ d_H(X, Y) = \max \left\{ \sup_{x \in X} \inf_{y \in Y} d(x, y), \sup_{y \in Y} \inf_{x \in X} d(x, y) \right\} \]
Hausdorff distance

• One-sided Hausdorff distance

\[ \text{Hausdorff}_1(A, B) = \max_{a \in A} \min_{b \in B} |a - b| \]

• Average Hausdorff (take one)

\[ \text{Hausdorff}_{\text{Ave}}(A, B) = \frac{1}{2} \text{Hausdorff}_1(A, B) + \frac{1}{2} \text{Hausdorff}_1(B, A) \]
Hausdorff distance

• One-sided Average distance

\[ \text{Hausdorff}_{1,Ave}(A, B) = \frac{1}{|A|} \sum_{a \in A} \min_{b \in B} |a - b| \]

• Average Hausdorff (take two)

\[ \text{Hausdorff}_{Ave}(A, B) = \max \left( \text{Hausdorff}_{1,Ave}(A, B), \text{Hausdorff}_{1,Ave}(B, A) \right) \]

• Or is it the average of the two instead of max??
Hausdorff distance

- One-sided fractional (95%) Hausdorff

\[ \text{Hausdorff}_{1.95}(A, B) = P_{95} \left( \min_{a \in A} \min_{b \in B} |a - b| \right) \]
Hausdorff distance

• One-sided fractional (95%) Hausdorff

\[ \text{Hausdorff}_{1,95}(A, B) = P_{95}\left( \min_{a \in A} |a - b| \right) \]

• Fractional (95%) Hausdorff

\[ \text{Hausdorff}_{95}(A, B) = \max \left( \begin{array} \text{Hausdorff}_{1,95}(A, B) \\ \text{Hausdorff}_{1,95}(B, A) \end{array} \right) \]
Hausdorff distance

• One-sided fractional (95%) Hausdorff

\[ \text{Hausdorff}_{1,95}(A, B) = P_{95} \left( \min_{a \in A} \min_{b \in B} |a - b| \right) \]

• Fractional (95%) Hausdorff

\[ \text{Hausdorff}_{95}(A, B) = \max \left( \text{Hausdorff}_{1,95}(A, B), \text{Hausdorff}_{1,95}(B, A) \right) \]

• Or is it the average?

• Or should I combine the points, then take 95%?
Here be dragons
Boundary Hausdorff

- Hausdorff distance may be computed on the set or the set boundary.
Boundary Hausdorff

- The max distance is to a point on the boundary
- So no difference, right?
Boundary Hausdorff

- The max distance is to a point on the boundary
- So no difference, right?

- Average distance will change
  \[ \text{Hausdorff}_{\text{Ave}}(\partial A, \partial B) = \cdots \]

- 95% distance will change
  \[ \text{Hausdorff}_{95}(\partial A, \partial B) = \cdots \]
Boundary Hausdorff

- "max" Hausdorff changes too

\[ \text{Hausdorff} \left( A, B \right) \neq \text{Hausdorff} \left( \partial A, \partial B \right) \]
Boundary Hausdorff

- "max" Hausdorff changes too

\[ \text{Hausdorff} \left( A, B \right) \neq \text{Hausdorff} \left( \partial A, \partial B \right) \]
Don't try this at home!
Di-HaRD

**Dice – Hausdorff Revolutionary Distance measure**

\[ \text{DiHaRD}(A, B) = \alpha \text{Dice}(A, B) + \beta \text{Hausdorff}(A, B) + \gamma \text{Hausdorff}(\partial A, \partial B) + \theta \text{Hausdorff}_{95}(A, B) + \nu \text{Hausdorff}_{95}(\partial A, \partial B) + \ldots \]