Registration Methods
Reference Material

- ITK Software Guide
  - Chapter 8

- Lecture Notes (Rensselaer / CenSSIS ERC)
  - http://www.cs.rpi.edu/courses/spring04/imagereg/

- ** Extra information can be found in the book: 
  - Insight Into Images (Yoo Book) Chapter 10
Overview

- Image Resampling
- Registration Framework
- Multi-Modality
Image Resampling
Why Resampling?

Resampling is the Essence of Intensity-Based Image Registration.
What is an Image?

An Image is a sampling of a continuous field using a discrete grid.
Image Origin & Spacing

Spacing (Sx)

Spacing (Sy)

Origin (Ox, Oy)
Index to Physical Coordinates

**** ASSUMING IDENTITY DIRECTION ****

\[ P[0] = \text{Index}[0] \times \text{Spacing}[0] + \text{Origin}[0] \]
\[ P[1] = \text{Index}[1] \times \text{Spacing}[1] + \text{Origin}[1] \]

\[ \text{Index}[0] = \text{floor} \left( \frac{P[0] - \text{Origin}[0]}{\text{Spacing}[0]} + 0.5 \right) \]
\[ \text{Index}[1] = \text{floor} \left( \frac{P[1] - \text{Origin}[1]}{\text{Spacing}[1]} + 0.5 \right) \]
Index to Physical Coordinates

With Direction Matrix

$$\tilde{\text{PhysicalLocation}} = \tilde{\text{Index}} \times \hat{\text{Spacing}} \times \hat{\text{Direction}} + \tilde{\text{Origin}}$$

$$\text{Index} = (\hat{\text{Spacing}} \times \hat{\text{Direction}})^{-1} \times (\tilde{\text{PhysicalLocation}} - \tilde{\text{Origin}})$$
Basic Resampling

Resampling

Trivial Cases
Sub-Sampling by Half

Origin \((O_x, O_y)\)

Spacing \((S_x)\)

Spacing \((2 \times S_x)\)

Spacing \((S_y)\)

Spacing \((2 \times S_y)\)

Image Region
Sub-Sampling by Half

Origin $(O_x,O_y)$

New Origin $(O'_x,O'_y)$

New Spacing $S'_x$

New Spacing $S'_y$
Super-Sampling by Double Origin (Ox, Oy)

Spacing (Sx)

Image Region

Spacing (Sx/2)

Spacing (Sy/2)

Spacing (Sy)

Origin (Ox, Oy)
Super-Sampling by Double Origin (O', O')

New Spacing S'x

New Spacing S'y

New Origin (O', O')

Origin (O, O)
Resampling in ITK

Input Image → Resample Filter → Output Image

- Transform
- Spacing
- Origin
- Region Start
- Region Size
- Interpolator
Coordinate System Conversions

Image Grid

Space Transform

Physical Coordinates

Image Grid

Physical Coordinates
Things I will not do...

I will not register images in pixel space
I will not register images in pixel space
I will not register images in pixel space
I will not register images in pixel space
I will not register images in pixel space
I will not register images in pixel space
I will not register images in pixel space
Fixed Image & Moving Image

Fixed Image Grid

Moving Image Grid

Fixed Image Physical Coordinates

Moving Image Physical Coordinates

Space Transform
Registration Framework
Components

Registration Method

Fixed Image

Moving Image

Interpolator

Transform

Metric

Optimizer
Image Metrics

- Mean Squares
- Normalized Correlation
- Mean Reciprocal Square Difference
- Mutual Information
  - Viola-Wells
  - Mattes
  - Histogram based
  - Histogram normalized
Transforms

- Translation
- Scaling
- Rotation
- Rigid3D
- Rigid2D
- Affine
- BSplines
- Splines: TPS, EBS, VS
Optimizers

- Gradient Descent
- Regular Step Gradient Descent
- Conjugate Gradient
- Levenberg-Marquardt
- One plus One Evolutionary Algorithm
Interpolators

- Nearest Neighbor
- Linear
- BSpline
Image Metrics

How similar is image A to image B?
Image Metrics

Does Image B match Image A better than Image C?
Image Metrics

\[ \text{Match}(A, B) > \text{Match}(A, C) \]
Image Metrics

Match( A, B )

Simplest Metric

Mean Squared Differences
Mean Squared Differences

For each pixel in A

\[ \text{Difference}(\text{index}) = A(\text{index}) - B(\text{index}) \]

\[ \text{Sum} += \text{Difference}(\text{index})^2 \]

\[ \text{Match}(A, B) = \frac{\text{Sum}}{\text{numberOfPixels}} \]
For each pixel in the Fixed Image

Fixed Image Grid

Moving Image Grid

Space Transform

Fixed Image Physical Coordinates

Moving Image Physical Coordinates
Evaluating many matches

Fixed Image

Moving Image

Transform
Plotting the Metric
Mean Squared Differences

Transform Parametric Space
Multi - Modality
Multi-Modality Registration
Multiple Image Modalities

Number of pairs
Multiple Image Modalities

More possible pairs
M.I. for Image Registration

Reference Image

Joint entropy = 5.53, M.I. = 5.53 [I(A,A) = H(A)]
M.I. for Image Registration

reference image

current image

difference image

joint entropy = 7.48  M.I. = 3.59
M.I. for Image Registration

Reference Image

Current Image

Difference Image

Joint Entropy = 9.36  M.I. = 1.70
Register T1 & T2